

## QCD Flux Tubes as Sigma Model Relics

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### Abstract

We describe flux tubes and their interactions in a low energy sigma model induced by  $SU(N_F) \rightarrow SO(N_F)$  flavor symmetry breaking in  $SO(N_c)$  QCD. Gauge confinement manifests itself in the low energy theory through flux tube interactions with unscreened sources. The flux tubes mediating confinement act as Alice strings in their cores, a phenomenon which may occur for  $\pi_2$ -line defects in physically realized systems.

Nonlinear sigma models successfully describe many low energy QCD phenomena. However, they have not captured the hallmark feature of QCD: confinement, where potentials between  $q\bar{q}$  pairs grow linearly with separation. Such linear potentials arise from flux tubes between unscreened  $q$  and  $\bar{q}$  sources for pure Yang-Mills QCD. Conventional sigma models describe the low-energy dynamics of QCD due to the global flavor symmetry breaking  $SU(N_F)_L \times SU(N_F)_R \rightarrow SU(N_F)_{diag}$ . Global flux tubes do not occur in these models, as the vacuum manifold  $G/H$  forms a Lie group, with trivial  $\pi_2$ . This absence of flux tubes is consistent with the absence of unscreened sources in QCD with fundamental quarks.

Witten [1] noted that an  $SO(N_c)$  gauge theory of QCD induces a different sigma model, whose topology *can* support flux tubes. The theory has  $N_F$  left-handed quarks which transform as (real) fundamentals of  $SO(N_c)$ . The color singlet condensate  $\langle q_{Li}^\alpha q_{Lj}^\alpha \rangle$  (where  $\alpha$  and  $i$  are color and flavor indices respectively) breaks the  $SU(N_F)$  flavor symmetry to its  $SO(N_F)$  subgroup. There is a baryon number  $SO(N_c)^2$  anomaly which leaves an anomaly free conserved  $Z_2$  baryon number. The Goldstone modes are described by a  $SU(N_F)/SO(N_F)$  nonlinear sigma model, with skyrmions ( $\pi_3(G/H) = Z_2$ , for  $N_F \geq 4$ ) and flux tubes ( $\pi_2(G/H) = Z_2$ , for  $N_F \geq 3$ ). This defect classification makes sense: (a) baryons are identified with antibaryons since real quarks are identified with antiquarks; (b) there are  $Z_2$  flux tubes which confine spinor sources, since only an even number of spinors can be screened by fundamental quarks.

In this letter we construct the flux tubes in this theory and show that their interactions with skyrmions and spinor sources obey heuristic expectations. We show that confinement in an  $SO(N_c)$  gauge theory can manifest itself in the low energy sigma model through flux tube interactions with unscreened sources. We proceed as follows: we derive the unique flux tube form with minimal energy; examine its classical stability and dynamics; and then study its quantum stability, spectrum, and interactions. We discuss how flux tubes act as Alice strings in their cores, form loops that support skyrmion number, and mediate the confinement of spinor sources. We skip many technical details, which appear in [2].

## Finding Nontrivial Flux Tubes

The sigma model for  $SO(N_c)$  QCD describes the dynamics of a sigma field  $\Sigma$ , which encodes the orientation of the fermion condensate  $\langle q_{Li}^\alpha q_{Lj}^\alpha \rangle$  with respect to its standard form  $\propto \delta_{ij}$ , with  $\Sigma = 1$ . Under an  $SU(N_F)$  transformation by  $g$ ,  $\Sigma$  transforms as  $\Sigma \rightarrow g\Sigma g^T$ , so that a generic  $\Sigma$  has the form  $gg^T$  for some group element  $g$ .  $\Sigma = \mathbb{1}$  is invariant under a  $g$  transformation if  $g \in SO(N_F)$ , so  $\Sigma$  is an element of the coset space  $G/H = SU(N_F)/SO(N_F)$ , which is not a Lie group.

$G$  invariance and cylindrical symmetry fixes the form of the minimal energy flux tube to be  $\Sigma = g(\theta)\Sigma_o(r)g^T(\theta)$  — an  $r$ -dependent vev, with angle-dependent group rotation, where  $(r, \theta, z)$  are cylindrical coordinates. Choosing  $\Sigma_o = \mathbb{1}$  at spatial infinity forces  $g(\theta)$  to lie in  $H = SO(N_F)$ , and to commute with  $\Sigma_o(r=0)$ , for a nonsingular solution with finite energy.

$\Sigma$  can be further specified using basis generators of the  $su(N_F)$  Lie Algebra, with Cartan norm  $\text{tr } T_a T_b = \frac{1}{2}\delta_{ab}$ . The rank  $N_F - 1$  Cartan subalgebra has as a basis diagonal matrices  $T_d$ ,

$$T_d = (2d(d+1))^{-1/2} \text{diag}(1, \dots, 1, -d, 0, \dots, 0), \quad (1)$$

with ones in the first  $d$  entries and  $1 \leq d \leq N_F - 1$ . We can overspecify them, sacrificing the Cartan norm, by Pauli matrices  $\{\frac{1}{2}\tau_{z(jk)}\}$  in all  $(jk)$  subplanes of the  $N_F$  dimensional vector space. Off-diagonal generators  $\{\frac{1}{2}\tau_{x(jk)}, \frac{1}{2}\tau_{y(jk)}\}$  complete the basis. The antisymmetric matrices  $\{T_h\} \equiv \{\frac{1}{2}\tau_{y(jk)}\}$  generate the unbroken  $H = SO(N_F)$  symmetry, while the symmetric matrices  $\{T_b\} \equiv \{\frac{1}{2}\tau_{x(jk)}, \frac{1}{2}\tau_{z(jk)}\}$  are broken generators.  $\Sigma_o(r)$ , a unitary symmetric matrix, can be written as  $\exp\{iF_b(r)T_b\}$ . Thus  $\Sigma$  assumes the most general nonsingular form  $\Sigma = h(\theta) \exp\{iF_b(r)T_b\}h^{-1}(\theta)$ , for  $h(\theta) \in H$  and  $F_b(r)$  ranging from zero at infinity to  $2\pi n \delta_{bb'}$  (for some direction  $b'$ ) at the origin.

We construct a non-trivial flux tube  $\Sigma$  from the exact sequence

$$\pi_2(SU(N_F)) = 0 \rightarrow \pi_2(SU(N_F)/SO(N_F)) \rightarrow \pi_1(SO(N_F)) = Z_2 \rightarrow \pi_1(SU(N_F)) = 0.$$

That is,  $gg^T$  gives a nontrivial  $\Sigma$  only if  $g$  corresponds to some mapping from the plane to  $SU(N_F)$ , with boundary values in the  $SO(N_F)$  subgroup. Furthermore, when parametrized

as a family of loops,  $g$  must start at the identity and end on a nontrivial loop in  $SO(N_F)$ . Taking  $(\alpha \in [0, 2\pi], \beta \in [0, \pi])$  as our coordinates on the plane, these criteria become

$$g(\alpha, \beta) = \begin{cases} \mathbb{1} & \text{when } \alpha = 0, \alpha = 2\pi, \text{ or } \beta = 0 \\ h^2(\alpha) & \text{when } \beta = \pi, \end{cases} \quad (2)$$

where  $h^2(\alpha)$  is a nontrivial loop in  $SO(N_F)$ .

Through technical arguments, two of us show that such trivializations of  $SO(N_F)$  loops in  $SU(N_F)$  have minimal energy only when they induce  $\Sigma$  of a very limited form [2]. To minimize energy,  $h^2(\alpha)$  is geodesic:  $h(\alpha) = \exp \{i\alpha T_{h\Box}\}$ , where  $T_{h\Box} = \{\frac{1}{2}\tau_y\}$  in some plane  $\Box$  — taken here as (12) for concreteness. This geodesic loop determines the associated flux tube  $\Sigma$ :

$$\begin{aligned} \Sigma(r, \theta) &= h(\theta) b(r) h^{-1}(\theta), \quad \text{with} \\ h(\theta) &= \exp \{i\theta T_{h\Box}\}, \quad b(r) = \exp \{iF_d(r)T_d\}, \\ \text{and } F_1(r=0) &= 2\pi; \quad F_d(r \rightarrow \infty) = 0. \end{aligned} \quad (3)$$

Here  $T_{h\Box}$  generates the loop  $h^2(\alpha)$ , and  $T_d$  are the Cartan generators from eq. (1). The boundary conditions on  $F_d$  stem from eq. (2), requiring  $g(\alpha, \beta)$  to trivialize  $h^2(\alpha)$ .

Note that when  $F_{d>1}$  vanishes,  $\Sigma$  lies entirely within a planar  $SU(2)$  subgroup of  $SU(N_F)$ , as  $T_1 = \tau_{z\Box}/2$ . This produces the simple form

$$\Sigma_{\Box}(r, \theta) = \mathbb{1} + (\cos(F/2) - 1) \mathbb{1}_{\Box} + i \sin(F/2) (\cos \theta \tau_{z\Box} - \sin \theta \tau_{x\Box}). \quad (4)$$

where  $F \equiv F_1$ ,  $\mathbb{1}_{\Box}$  gives the identity in the plane  $\Box$  and vanishes outside it, and  $\mathbb{1}$  is the usual  $SU(N_F)$  identity. In this letter, we consider only such planar flux tubes, having shown in [2] that they minimize energy for the action discussed here.

### Flux Tube Stability and Dynamics

As a minimal model for  $\Sigma$  with stable skyrmions, consider the Skyrme lagrangian [5] \*

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\*We use QCD sigma model parameters  $e, F_\pi, m_\pi$ , etc., although these are not the usual pions. There is also a Wess-Zumino term, which vanishes for constant- $z$  flux tubes.

$$\mathcal{L}_0 = \frac{F_\pi^2}{16} \text{tr} \partial_\mu \Sigma \partial^\mu \Sigma^\dagger + \frac{1}{32e^2} \text{tr} [\Sigma^\dagger \partial_\mu \Sigma, \Sigma^\dagger \partial_\nu \Sigma]^2 . \quad (5)$$

This model does not have stable flux tubes, for under the rescaling  $\Sigma(r, \theta, z) \rightarrow \Sigma(\lambda r, \theta, z)$ , the tension of a finite flux tube can decrease. Flux tubes diffuse to infinite size ( $\lambda = 0$ ) to lower their energy.

There are several modifications to the minimal  $\mathcal{L}_0$ , or to the minimal form for  $\Sigma$ , that lead to stable flux tubes [2]. One natural choice is to give non-zero mass to the quarks in the original QCD theory. This induces a mass term in the effective lagrangian,

$$\mathcal{L}_m = \frac{F_\pi^2 m_\pi^2}{16} \text{tr} (\Sigma + \Sigma^\dagger - 2 \cdot \mathbb{1}) , \quad (6)$$

which gives mass to the Goldstone bosons, and also stabilizes the flux tube solutions.

The Skyrme action eq. (5) gives gradient energy density to the flux tube:

$$\rho_0 = \frac{F_\pi^2}{16} \text{tr} \left\{ (F'_d T_d)^2 + \frac{1}{r^2} (\tilde{T}^2 - [F'_d T_d, \tilde{T}]^2) \right\} , \quad (7)$$

where

$$\tilde{T} \equiv b^{-1}(r) [T_{h\Box} , b(r)] , \quad (8)$$

and  $r$  has been rescaled to dimensionless units  $e F_\pi r_{phys}$ . For the planar solution eq. (4), this gives the energy density

$$\rho_0 = \frac{F_\pi^2}{16} \left\{ \frac{1}{2} F'^2 + \frac{1}{r^2} (1 - \cos F) (1 + F'^2) \right\} . \quad (9)$$

To this term we add potential energy from the pion mass term eq. (6),

$$\rho_m = \frac{\lambda^2 F_\pi^2}{16} (1 - \cos(F/2)) , \quad (10)$$

where  $\lambda = 2m_\pi/e F_\pi$ . Together, these energy contributions determine a nonlinear equation of motion for  $F$ , which we solve numerically. Solutions appear in Figure 1 for different values of  $\lambda$ , including the physical  $\lambda_0 = 0.236$  ( $e = 2\pi$ ,  $m_\pi = 138$  Mev and  $F_\pi = 186$  Mev). Increasing  $\lambda$  raises the flux tube's energy density while shrinking its core size. Inside the

core,  $F$  falls linearly from  $F(0) = 2\pi$ ; outside, it scales as the hyperbolic Bessel function  $F \sim x^{-1/2} \exp\{-x\}$ , with  $x = \lambda r/2$ .  $2\lambda^{-1}$  sets the flux tube's dimensionless core size, which gives physical core size  $m_\pi^{-1}$ . The flux tube tension is proportional to  $F_\pi^2$  — which is proportional to the number of colors  $N_c$  in the large  $N_c$  limit. This supports its identification with the confining force between spinor sources, since the Casimir for  $SO(N_c)$  spinors is of order  $N_c$ . Numerically we find tension  $4.6F_\pi^2$  when  $\lambda = \lambda_0$ , scaling linearly with  $\lambda$ .

### Quantum Stability and Spectrum

The quantum numbers and low-lying internal excitations of the flux tube are given by quantizing the zero-mode collective coordinates [3],

$$\Sigma(t, r, \theta) = A(t) \Sigma(r, \theta) A^{-1}(t) , \quad (11)$$

where  $A(t)$  rotates about  $n_h T_h$  with dimensionless frequency  $\omega$ .<sup>†</sup> These modes have rotational energy confined to the string core, which can be calculated from the Skyrme action eq. (5):

$$\rho_\omega = \frac{F_\pi^2 \omega^2}{16} \text{tr} \left( \hat{T}^2 - [F'_d T_d, \hat{T}]^2 - \frac{1}{r^2} [\tilde{T}, \hat{T}]^2 \right) . \quad (12)$$

with  $\tilde{T}$  from eq. (8) and

$$\hat{T} \equiv b^{-1}(r) [ h^{-1}(\theta) n_h T_h h(\theta) , b(r) ] . \quad (13)$$

Calculating eq. (12) is tedious, and described in [2]. We show there that the planar vacuum survives quantum fluctuations due to zero modes. It has the classical two dimensional Lagrange density

$$L(\tilde{z}, t) = -\tilde{\rho} - \frac{\Lambda_\square}{2} (\text{tr} A^\dagger \dot{A} T_{h_\square})^2 - \frac{\Lambda_{\square'}}{2} \sum_{\square'} (\text{tr} A^\dagger \dot{A} T_{h_{\square'}})^2, \quad (14)$$

where  $\tilde{z}$  measures dimensionless length along the flux tube and the planes  $\square'$  intersect  $\square$  in single lines. Our numerical solutions, modified by rotational back reaction, give for the

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<sup>†</sup>We neglect other excitations, such as bending modes.

integrals  $\tilde{\rho} \approx 5F_\pi/e$ ,  $\Lambda_\square \approx 40/e^3 F_\pi$ , and  $\Lambda_{\square'} \approx 16/e^3 F_\pi$ . The moments of inertia  $\Lambda_\square$  and  $\Lambda_{\square'}$  vary little with back reaction for quantum excited states; however, the tension  $\tilde{\rho}$  grows linearly with  $\omega$ , due to compression of the rotating flux tube.

To quantize this Lagrangian, we must rewrite the time derivatives in eq. (14) in terms of canonical momenta. Under an  $SO(N_F)$  transformation  $h$ ,  $\Sigma \rightarrow h\Sigma h^T$ , so that  $A \rightarrow hA$  from eq. (11). The generators  $\mathcal{I}_h$  of  $H$  rotations thus generate left-transformations of  $A$ . Similarly, we define body-centered generators  $\mathcal{I}'_h$  generating right transformations,  $A \rightarrow Ah$ .  $\mathcal{I}_h$  and  $\mathcal{I}'_h$  are related by an orthogonal transformation,

$$\mathcal{I}_h = R_{hh'} \mathcal{I}'_{h'}, \quad R_{hh'} = 2 \operatorname{tr} \left( A^\dagger T_h A T_{h'} \right) . \quad (15)$$

Noether's theorem then gives for  $\mathcal{I}'_h$ ,

$$\mathcal{I}'_h = i\Lambda_h \operatorname{tr} (A^\dagger \dot{A} T_h) \quad (\text{no sum}). \quad (16)$$

The flux tube solution  $\Sigma(r, \theta)$  is invariant under the action of  $\mathcal{I}'_\square + J_z$ , so that  $\mathcal{I}'_\square$  is  $-J_z$  (much like the Skyrme model, where  $\vec{I}$  in the body centered frame is  $-\vec{J}$ ). It is also invariant under  $SO(N_F - 2) \subset SO(N_F)$ , acting on the subspace  $\square_\perp$  orthogonal to  $\square$ . This restricts  $\Sigma$  to quantum states  $\mathcal{I}'$  containing a singlet under  $SO(N_F - 2)$ .

Given these constraints, we write the Hamiltonian density obtained from eq. (14) in terms of the physically observable Noether charges,  $\mathcal{I}_h$  and  $J$ ,

$$H(\tilde{z}, t) = \tilde{\rho} + \frac{1}{2\Lambda_\square} J_z^2 + \frac{1}{2\Lambda_{\square'}} (\mathcal{I}^2 - J_z^2) , \quad (17)$$

which is quantized subject to the constraint  $\mathcal{I}'_\square = -J_z$ , with bosonic defects for  $N_F > 3$ . For three flavors, this gives a spectrum of states  $(I, J_z)$  where  $I$  and  $|J_z|$  are integers,  $-I \leq J_z \leq I$ , and  $\mathcal{I}^2 = I(I+1)$ .

### Flux Tube Interactions

Our sigma model has both flux tube and skyrmion solutions. We show that flux tube solutions are Alice strings. Moreover, loops of flux tube can carry skyrmion number, just as loops of gauged Alice string support monopole charge. We construct the skyrmions

and discuss their interactions with flux tubes, showing that only the topologically trivial combination of two flux tubes can end on a skyrmion. This suggests that, while baryons (i.e. skyrmions) are not confined, the spinor sources which combine to form them are, with confinement mediated by  $Z_2$  flux tubes joining them.

First we consider twisted flux tubes. Our flux tubes have the Alice property: some unbroken symmetries preserve a local vev but cannot be extended globally, since transport around the string makes them multivalued [4]. This property seems ambiguous, as the flux tube eq. (3) allows many choices of underlying  $g(r, \theta)$ . Double-valued choices — such as  $g(r, \theta) = h(\theta) b(F(r)/2)$  — give unbroken generators  $g T_{h\Box'} g^{-1}$  double-valued in  $\theta$ ; while single-valued choices — such as  $g(r, \theta) = h(\theta) b(F(r)/2) h(\theta)$  — leave all generators single-valued. This ambiguity persists even when we recast the flavor-dependent quark mass  $\Sigma$  as an interaction between flavor gauge fields  $g^{-1}(r, \theta) \partial_\mu g(r, \theta)$  and shifted quarks  $Q_L = g(r, \theta) q_L$ . [3] This identifies the flux tube with a gauged string, with Wilson loop  $U(2\pi) = \text{P exp} \left( \oint \vec{A} \cdot d\vec{l} \right)$ . However, the ambiguity persists:  $g(r, \theta)$  double-valued gives Wilson loop  $U(2\pi) = 1 - 2 \mathbb{1}_\Box$ , making both quarks  $Q_L$  and generators  $T_{h\Box'}$  double-valued. Single-valued  $g(r, \theta)$  produces instead single-valued quarks with an  $r$ -dependent Wilson loop, giving  $U(2\pi) = 1$  and single-valued generators at  $r = \infty$ , but nontrivial  $U(2\pi)$  and multi-valued generators  $T_{h\Box'}$  at finite  $r$ .

This ambiguity can be resolved physically, by considering adiabatic transport of quarks around the flux tube. [6] Under such transport, quarks remain in their mass eigenstates. These are fixed by two terms: a flavor-independent bare Majorana mass  $M$  (breaking  $SU(N_f) \rightarrow SO(N_f)$  explicitly) and the flavor-dependent Majorana mass  $\mu\Sigma$ , where  $\mu$  is the  $SU(N_f)$ -breaking vev. These give mass eigenstates double-valued in  $\theta$ . However, the states have mass splitting  $\Delta m^2 = -2iM(\mu - \mu^*) \sin(F/2)$ , with  $M$  real. Thus the quarks are degenerate, and unaffected by transport around the string, unless the vev  $\mu$  misaligns in phase with the bare mass  $M$ . In that case, quarks at finite radius have double-valued wave functions and Aharonov-Bohm scatter off the flux tube, flipping the sign of their  $T_{h\Box'}$



charges.<sup>‡</sup> Thus the flux tube acts as an Alice string in its core. This analysis readily generalizes to other  $\pi_2$ -strings. This suggests the promise of observing similar effects in nonabelian physical systems with global  $\pi_2$ -strings, such as liquid He-3a. [7]

Since twisted Alice loops can support monopole charge, we calculate the skyrmion number of a twisted flux loop,  $\Sigma(z, r, \theta) = A(z) \Sigma(r, \theta) A^{-1}(z)$  with  $A(z) = \exp(iz l n_h T_h)$ .  $2\pi$ -periodicity implies that  $l$  is integral for planar  $n_h = n_\square$ , and even otherwise. Thus any twisted flux loop can deform to the planar flux loop  $\Sigma(z, r, \theta) = h(\theta + lz) b(F(r)) h^{-1}(\theta + lz)$ , taking values within the subspace  $SU(2)/SO(2) \sim S^2$ . Thus we can identify its  $\pi_3$  index with the map's Hopf number, *i.e.* the linking number between any two fibers of constant  $\Sigma$  in physical space. From [8], this linking number is precisely  $l$  — the number of times a nontrivial fiber  $\Sigma_0$  twists around the loop's core, which has  $\Sigma = -\mathbb{1}_\square$ . Thus flux loops with an  $l = 1$  planar twist form fundamental skyrmions; flux loops with nonplanar twist are trivial for  $N_f > 3$ .

A nicer parametrization of the skyrmion stems from the exact sequence

$$\pi_3(SO(N_f)) \rightarrow \pi_3(SU(N_f)) \rightarrow \pi_3(SU(N_f)/SO(N_f)) \rightarrow \pi_2(SO(N_f)) = 0 .$$

This identifies skyrmions on the vacuum manifold with images of skyrmions in  $SU(N_f)$ ,  $g(r, \hat{n}) = \exp(iF_s(r) \hat{n}_i T_{i\square})$ . (Here  $r$  and  $\hat{n}$  are the radius and unit direction vector in 3-space, and  $F_s(r)$  approaches  $2\pi$  at  $r = 0$  and zero at  $r = \infty$ .) This gives an axisymmetric skyrmion

$$\begin{aligned} \Sigma_s = & \mathbb{1} + (\cos F_s - 1) (1 - n_z^2) \mathbb{1}_\square \\ & + i \left( \sin F_s (n_y \tau_{x\square} + n_x \tau_{z\square}) + 2 \sin^2(F_s/2) n_z (-n_x \tau_{x\square} + n_y \tau_{z\square}) \right) , \end{aligned} \quad (18)$$

after global spatial rotation. Equation (4) for the flux tube lets us identify the angular winding of a flux tube with that of a skyrmion in the  $xy$ -plane ( $n_z = 0$ ). Thus, if their

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<sup>‡</sup>We treat quarks with trajectories well-separated from the origin. This holds asymptotically inside the flux tube, where  $m_i^{-1} < r < m_\pi^{-1}$ , for quark mass eigenvalues  $m_i$ .

radial forms coincided, we could deform the skyrmion's lower hemisphere into a flux tube. However, since both  $F$  and  $F_s$  vary from 0 to  $2\pi$ , the skyrmion cannot end in a single flux tube. Instead it joins only to flux tubes where  $F(r)$  ranges from 0 to  $4\pi$  — that is, configurations with two flux tubes, which are trivial. Thus skyrmions are not confined.

However, objects which combine to form skyrmions can interact with the flux tubes. Such “half-skyrmions” could arise as external spinor sources in the underlying theory. They are confined, as fundamentals cannot screen them. As mappings on  $G/H$ , they appear precisely as half-skyrmions — objects of form (18) with  $F_s(r)$  ranging from 0 to  $\pi$ . Such objects are not defects in the conventional sense, since they have linearly divergent energy — like an unscreened point source. They can join to their opposite winding counterparts via single flux tubes, restricting linearly divergent energy to their separation distance. Thus confinement of sources in an  $SO(N_c)$  gauge theory persists in the low energy Skyrme model.

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## FIGURES

FIG. 1. a.) Flux tube solutions  $F(r)$  for  $\lambda$  values 0.2, 0.236, 1.0, and 2.0. The dotted line corresponds to the physical value  $\lambda_0 = 0.236$ . Core size shrinks with increasing  $\lambda$ . b.) The above flux tubes' energy density  $\rho$  ( $\rho_0 + \rho_m$ , from text). Tension grows with  $\lambda$ .

This figure "fig1-1.png" is available in "png" format from:

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